Notes: 8.3 Products & Quotients of Complex Numbers in Polar Form

Product of Complex Numbers in Polar Form

$$r_1(\cos\theta_1 + i\sin\theta_1) \cdot r_2(\cos\theta_2 + i\sin\theta_2)$$

$$= r_1 r_2\left[\cos(\theta_1 + \theta_2) + i\sin(\theta_1 + \theta_2)\right]$$

modulus ($r_1 r_2$)

argument ($\theta_1 + \theta_2$)

Quotient of Complex Numbers in Polar Form

$$\frac{r_1(\cos\theta_1 + i\sin\theta_1)}{r_2(\cos\theta_2 + i\sin\theta_2)} =$$

$$= \frac{r_1}{r_2}\left[\cos(\theta_1 - \theta_2) + i\sin(\theta_1 - \theta_2)\right]$$

modulus

argument

Powers and Roots of Complex Numbers

De Moivre's Theorem

$$\left[r\left(\cos\theta + i\sin\theta\right)\right]^{n} = r^{n}\left(\cos n\theta + i\sin n\theta\right)$$

POLAR FORM

Be careful with order of operations. ↑
Multiply n•θ first, then apply cosine & sine.

if $n = \text{integer}$, then you are finding a power

if $n = \text{fraction}$, then you are finding a root

Example: find the product

A. $5(\cos\pi + i\sin\pi)\cdot 2(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4})$

mod: $5(2) = 10$ (modulus)

arg: $\frac{4\pi}{4} + \frac{3\pi}{4} = \frac{7\pi}{4}$ (argument)

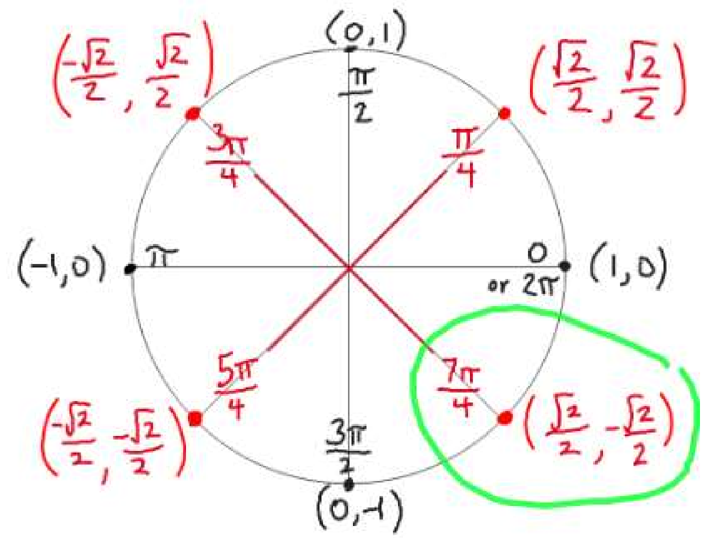
$10(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4})$

Your homework may ask you to stop here at polar form of a complex #

$= 10\left(\frac{\sqrt{2}}{2} + -\frac{\sqrt{2}}{2}i\right)$

$= 5\sqrt{2} - 5\sqrt{2}i$

Rectangular form of a complex #



Example: find the quotient

B. $$\frac{5(\cos\pi + i\sin\pi)}{2\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)} = \frac{5}{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$$

polar form of a complex #

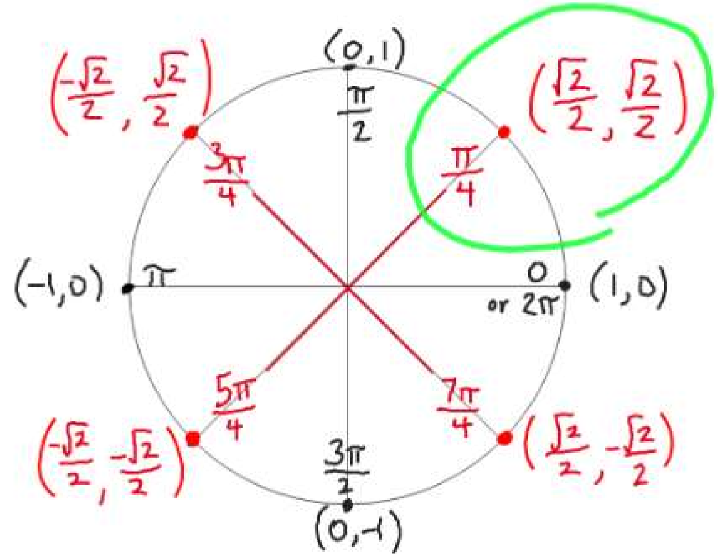
$$= \frac{5}{2}\left(\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}\right)$$

$$= \boxed{\frac{5\sqrt{2}}{4} + \frac{5\sqrt{2}}{4}i}$$

Rectangular form of a complex #

mod: $\frac{5}{2}$

arg: $\frac{4\pi}{4} - \frac{3\pi}{4} = \frac{\pi}{4}$



REMINDER:

Evaluate $(\sqrt{3} - i)^2$

$\rightarrow$ multiply $\left(\sqrt{3} - i\right)\left(\sqrt{3} - i\right)$

then simplify using $i^2 = -1$

$$\left(\sqrt{3} - i\right)\left(\sqrt{3} - i\right) = 3 - 2i\sqrt{3} + i^2$$
$$= 3 - 2i\sqrt{3} + -1$$
$$= 2 - 2i\sqrt{3}$$

This is a reasonable calculation to do "by hand."

Finding an expression to the 2nd power is somewhat reasonable by expansion…but what about $\left(\sqrt{3}-i\right)^{7}$ **or** $\left(\sqrt{3}-i\right)^{\frac{1}{3}}$ **???**

Power ↑ Root ↑

TOO HARD to calculate by hand!!!

Pascal's Triangle or Binomial Theorem could help us, but there would be many steps to simplify.

Use DeMoivre's Theorem instead.

Example:

$(3, \sqrt{3})$ Quad I → polar form

$$\left[r(\cos\theta + i\sin\theta)\right]^n = r^n(\cos n\theta + i\sin n\theta)$$

$a + bi$

C. $(3+\sqrt{3}\,i)^5$

$r = \sqrt{a^2+b^2}$

$r = \sqrt{3^2+(\sqrt{3})^2}$

$r = \sqrt{9+3}$

$r = \sqrt{12}$ $\quad \sqrt{4}\cdot\sqrt{3}$

$r = 2\sqrt{3}$

$\tan\theta = \frac{b}{a}$

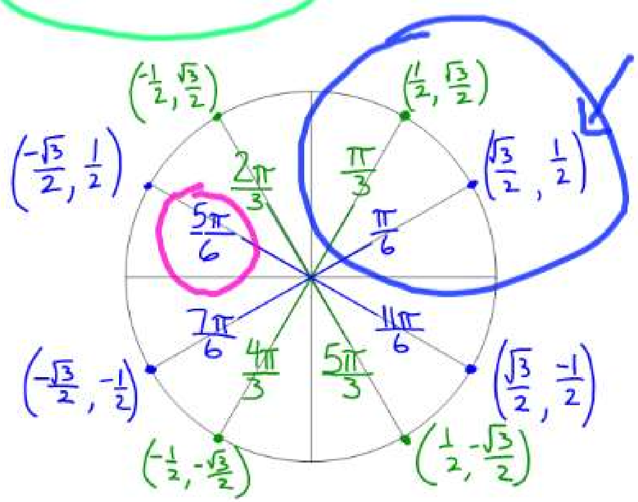
$\tan\theta = \frac{\sqrt{3}}{3}$

$\theta = \frac{\pi}{6}$

$$2\sqrt{3}\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)^5$$

Now simplify to get rectangular form.

See next slide…



$$\frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

Example:

$$\left[r(\cos\theta + i\sin\theta)\right]^n = r^n(\cos n\theta + i\sin n\theta)$$

C. $(3+\sqrt{3}\,i)^5$

Now simplify to get rectangular form→

$(3, \sqrt{3})$ Quad I → polar form

$a+bi$

$2\sqrt{3}\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)^5$

$= (2\sqrt{3})^5\left(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right)$

$= 2^5\sqrt{3}^5\left(-\frac{\sqrt{3}}{2} + \frac{1}{2}i\right)$

$= 32\cdot3\cdot3\cdot\sqrt{3}\left(-\frac{\sqrt{3}}{2} + \frac{1}{2}i\right)$

$= 288\sqrt{3}\left(-\frac{\sqrt{3}}{2} + \frac{1}{2}i\right)$

$= -432 + 144\sqrt{3}\,i$

CHECK 8.3 EVEN ANSWERS:

78. $2\left(\cos\frac{\pi}{18} + i\sin\frac{\pi}{18}\right)$

82. $2^{\frac{1}{6}}\left(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}\right)$

84. $\left(\cos\frac{\pi}{10} + i\sin\frac{\pi}{10}\right)$

Do NOT graph #78, 82, 84

Leave answer in polar form since not easily evaluated with unit circle.

8.3 HINTS:

(78) $(4\sqrt{3}+4i)^{\frac{1}{3}=n} = \left[r(\cos\theta + i\sin\theta)\right]^{\frac{1}{3}}$

$= r^{\frac{1}{3}}\left(\cos\tfrac{1}{3}\theta + i\sin\tfrac{1}{3}\theta\right)$

$r = \sqrt{}$

$\tan\theta =$

(84) $i^{\frac{1}{5}} \rightarrow (0 + 1i)^{\frac{1}{5}}$

($a = 0$, $b = 1$)

$r = \sqrt{}$

$\tan\theta =$